Name: Key

Math 221: Test 1 - 6/8/15

Do all work on the test below. Write down the methods you used when trying to solve the problems. You must show all work to receive full credit.

1. Matching: Match the following vocabulary words to their definitions on the right. Place the correct letter in the space provided. [1 Point Each]

G	1. Cardinal Number	A. The set of all elements being considered in a given discussion.
N	2. Complement	B. The number of cardinals appearing in the set.
I	3. Element	C. The set of elements which are in either of the referenced sets.
L	4. Empty Set	D. A set which is a subset of a second set but is not equal to it.
M	5. Intersection	E. Any collection of objects with no repetitions.
D	6. Proper Subset	F. A set in which each of its elements are also elements of a second set.
E	7. Set	G. The number of elements in a set.
K	8. Set Difference	H. A set which is smaller than a second set.
F	9. Subset	I. An object within a set.
C	10. Union	J. Any collection of objects, with repetitions allowed.
A	11. Universal Set	K. The set of elements which are in the first set but not in the second set.
		L. The set with no elements.
		M. The set of elements which are in both of the referenced sets.
		N. The set of elements in the universal set that are outside the referenced set.

2. Briefly **explain** each step of the 4 step problem solving process. You do not have to write the official names of each step if you explain it well. [8 Points]

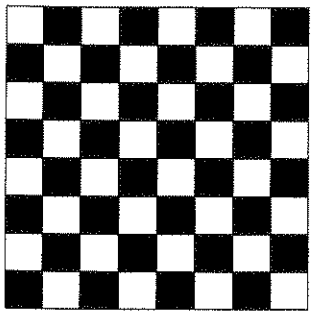
(1) Understand the Problem - Make sure that the problem makes sense, the variables and question are clear, and all terminology is understood.

(2) Devise a Plan - Strategies include looking at a simpler case, finding a pattern, or drawing a picture.

(3) Carry Out the Plan - Do each step of your plan, checking your work as you go.

(4) Look Back - Check your work and make sure you answered the question. Also, consider other strategies you could have used or similar problems that could be solved similarly.

3. How many squares can be found in the 8 by 8 checkerboard figure below? [9 Points]



1 by 1 squares: $8 \times 8 = 64$
2 by 2 " : $7 \times 7 = 49$
3 by 3 " : $6 \times 6 = 36$
4 by 4 " : $5 \times 5 = 25$
$4 \times 4 = 16$
$3 \times 3 = 9$
$2 \times 2 = 4$
8 by 8 squares: $1 \times 1 = 1$

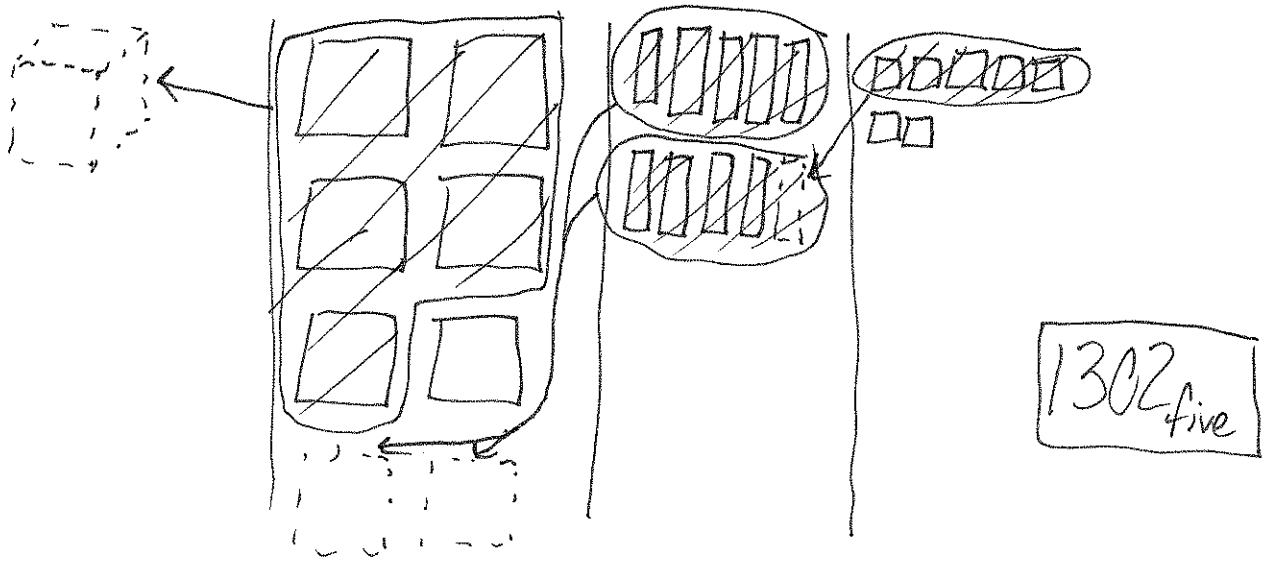
Adds up to $\boxed{204}$

4. Calculate the value of the sum $1 + 3 + 5 + ... + 41$. [6 Points]

$1+3+5+\ldots+41=S$
$41+39+37+\ldots+1=S$
$42+42+42+\ldots+42=2S$
21 numbers

$2S = 42(21)$
$S = \frac{42(21)}{2} = \boxed{441}$

5. Perform the exchanges to write the following situation as a number in base five: six flats, nine longs, and seven units. Either draw a picture of what you did or write down your steps with each of your exchanges. [9 Points]



$\boxed{1302_{five}}$

6. Fill in the blanks in the counting of base five numbers. Remember, if you're not sure how to go to the next step, try making exchanges when you put another unit in the picture. [1 Point Each]

1_{five}, 2_{five}, 3_{five}, 4_{five}, 10_{five}, 11_{five}, 12_{five}, 13_{five}, 14_{five}, 20_{five}, ..., 43_{five}, 44_{five},

100_{five}, ..., 143_{five}, 144_{five}, 200_{five}, ..., 443_{five}, 444_{five}, 1000_{five}.

7. Perform the following conversions. [6 Points Each]

(a) 1341_{five} to base ten.

$1(5^3)+3(5^2)+4(5^1)+1$

$=125+3(25)+20+1$

$=221_{\text{ten}}$

(b) 390_{ten} to base five.

$125\overline{)390}\ 3$

-375

$25\overline{)15}\ 0$

-0

$5\overline{)15}\ 3$

-15

0

3030_{five}

8. Given $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 3, 5, 7\}$, and $B = \{1, 4, 8\}$, find the following. [5 Points Each]

(a) $\overline{B} \cup A$

$\{2,3,5,6,7\} \cup \{1,3,5,7\}$

$= \{1,2,3,5,6,7\}$

(b) $\overline{A} \cap B$

$\{2,4,6,8\} \cap \{1,4,8\}$

$= \{4,8\}$

(c) $A - B$

irrelevant

$\{1,3,5,7\} - \{1,4,8\}$

$= \{3,5,7\}$

9. Fill in the following blanks with $\in$, $\notin$, $\subseteq$, or $\not\subseteq$. If the blank is filled with $\subseteq$, also tell whether you could also put $\subset$ or $=$. [4 Points Each]

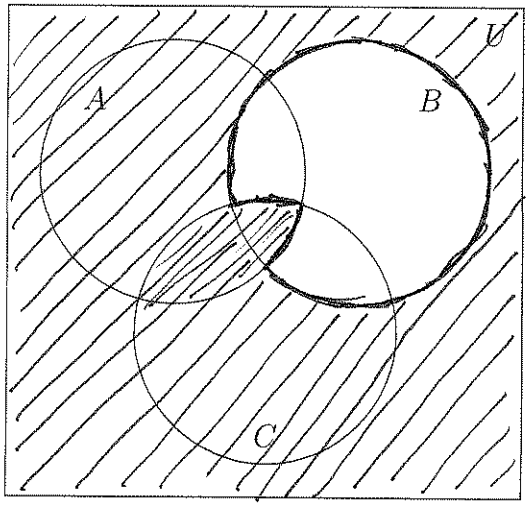
(a) $0 \underline{\notin} \varnothing$

(b) $\mathbb{Z} \underline{\subseteq, \subset} \mathbb{R}$

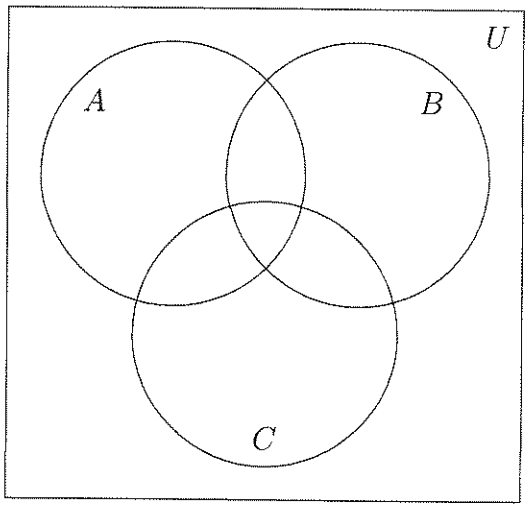
(c) $18 \underline{\notin} \{-2x \mid x \in \mathbb{N}\}$ $\{-2, -4, -6, -8, \ldots\}$

(d) $\{x^2 \mid x \in \mathbb{N}\} \underline{\subseteq, \subset} \{0, 1, 4, 9, 16, \ldots\}$ $\{1, 4, 9, 16, \ldots\}$

10. Represent $\overline{B} \cup (A \cap C)$ on a Venn Diagram. (Note: I put 2 diagrams on here in case you make a mistake and need another.) [9 Points]



Highlight everything shaded.



The following bonus questions should not be attempted until you have solved every other question and checked your answers. Please see me if you need extra paper.

Bonus 1: Write a formula for the number of toothpicks in the n^{th} stage of the pattern to the right. [5 Points]

1	2	3	...	n
4	$4 \cdot 2^2$	$4 \cdot 3^2$	...	$4n^2$

$4n^2$

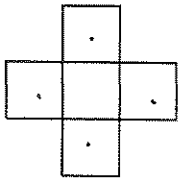


Figure 2

Figure 1

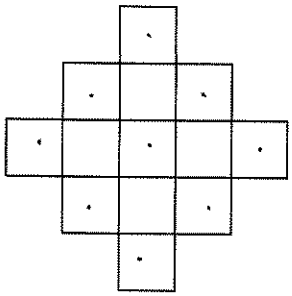


Figure 3

Bonus 2: Write a set that would have the figure to the right as its Venn Diagram. [5 Points]

$(A \cap B) \cup (A \cap C) \cup (B \cap C)$